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Torsional vibration suppression by wave absorption controller

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Abstract

We discuss vibration suppression in a lumped torsional system using a wave-absorption filter. The controlled system is that treated previously [M. Saigo, et al., Torsional vibration suppression by wave-absorption control with imaginary system, Journal of Sound and Vibration 270 (2004) 657–672] in which an imaginary wave-propagation system similar to the real controlled system is computed online. Here, we introduce a new control algorithm we developed, using a modified impedance-matching wave-absorption filter taking the boundary condition into consideration, and that requires no online computation of an imaginary system with initialization. Our new wave filter provides much better control performance than the imaginary system computed online. The filter's basic concept enables the dynamics of the end element to which a control actuator is connected to meet the wave solution by control force. Experiments for 2 and 3 dof systems confirm that the wave-absorption control filter realizes traveling wave characteristics accurately without reflecting and shows very high control performance in vibration suppression.

1. Introduction

Active vibration suppression in structures and mechanical systems consists primarily of modal vibration control based on natural vibration modes, i.e., the system's standing-wave state, and wave-absorption control based on vibration-energy absorption by reflection wave canceling, i.e., the system's progressive-wave state. Wave absorption control has advantages over modal vibration control, which is used widely in different fields. The wave-absorption process is conducted based on local wave-propagation properties, with no need to deal with a total system, and is applicable to any system even if only information on the structure where waves propagate is known. Wave-absorption control, however, requires information on where waves propagate, making it suitable for one-dimensional (1D) structures or assemblies of 1D elements. Wave control in 1D structures [1–11] has been widely studied in continuous structures such as beams and truss structures, but less in lumped systems [11–13]. O'Connor and Lang [11] used a lumped parameter spring-and-mass system to model a flexible arm, while Saigo et al. [12–14] studied a multiple-pendulums system and a multiple degree-of-freedom (dof) lumped torsional system.

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Nomenclature

K_i, K spring constant of ith torsional bar for nonuniform and uniform systems
I_i, I moment of inertia of ith disc for nonuniform and uniform systems
t time
T_i disturbance torque on ith disc
ϕ_i angle of ith torsional bar
ω_0 specific frequency for homogeneous system ($=\sqrt{K/I}$)

Online computer simulation of a large dof structural system having properties similar to the actual controlled system has been used to avoid treating an irrationonal transfer function inherently existed in wave propagation [12–14]. This "imaginary" system is connected virtually to the real system by an actuator satisfying the continuity condition between real and imaginary systems. This strategy realizes an infinite structural system free of wave reflections in the controlled real system if a suitable process is conducted to clear vibrating energy in the imaginary system at appropriate timing. When the imaginary system does not have enough degrees of freedom to absorb all vibration energy of the real system, the imaginary system will give the solution which reflects at the end of the imaginary system. This solution returns to the real system vibration energy absorbed from the real system before. For this, the process initializes the imaginary system where deflection and velocity of all elements are set to zero except for the end element of the connecting side. Initialization should be done before the reflecting wave from the end of the imaginary system reaches the real system. Control effectiveness in a lumped torsional system has been shown both theoretically and experimentally [14], but control performance in experiments was not satisfactory. This was due primarily to the initialization reaction, i.e., the successive connection of the initialized imaginary system to the real system makes a torque jump leading to computation error in the imaginary system due to suddenly increased amplitude.

We studied the impedance-matching transfer function of wave-absorption control to a lumped torsional system needing no initialization. The equation of motion of the end element of a rotating torsional system corresponds to that of a spring-and-mass system with a fixed boundary condition to which the usual impedance-matching condition cannot be applied. For this, we investigate two strategies—a new imaginary system having an impedance-matching condition at the end element and a modified impedance-matching condition directly applicable to the real end element of the fixed boundary condition. The characteristic wave solution, an irrational function, is approximated with fractional polynomials by curve fitting and used to make up a so-called IIR filter with impedance-matching condition. We also conducted experiments for 2 and 3 dof systems.

2. Control law

2.1. Equation of motion

The 1D torsional vibration system considered consists of torsional bars and rigid discs (Fig. 1). Rigid lines represent the real system and dotted lines the imaginary system. Our control compensates for torque $K_{m+1}\phi_{m+1}$, i.e., generated at the (imaginary) connecting torsional bar between real and imaginary discs, by an actuator, and absorbs vibration energy in the real system propagated to the imaginary system.

Consider a $(m+n)$-dof torsional vibration system in which the controlled (real) system is m-dof and the imaginary system n-dof. The equation of motion is expressed as

$$\ddot{\phi}_1 + \frac{K_1}{I_{0,1}}\phi_1 - \frac{K_2}{I_1}\phi_2 = -\frac{T_0}{I_0}$$

$$\ddot{\phi}_m - \frac{K_{m-1}}{I_{m-1}}\phi_{m-1} + \frac{K_m}{I_{m-1,m}}\phi_m - \underbrace{\frac{K_{m+1}}{I_m}\phi_{m+1}}_{a} = -\frac{T_{m-1}}{I_{m-1}} + \frac{T_m}{I_m}$$

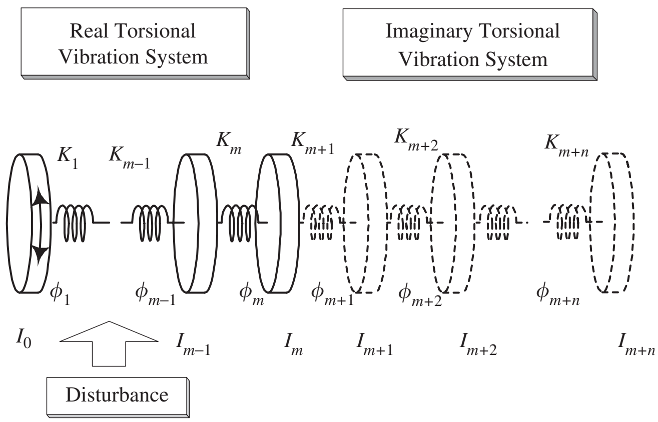

Fig. 1. Real and imaginary torsional vibration systems connected.

$$\ddot{\phi}_{m+1} - \frac{K_m}{I_m}\underbrace{\phi_m}_{b} + \frac{K_{m+1}}{I_{m,m+1}}\phi_{m+1} - \frac{K_{m+2}}{I_{m+1}}\phi_{m+2} = 0$$

$$\cdots\cdots$$

$$\ddot{\phi}_{m+n} - \frac{K_{m+n-1}}{I_{m+n-1}}\phi_{m+n-1} + \frac{K_{m+n}}{I_{m+n-1,m+n}}\phi_{m+n} = 0$$

$$\frac{1}{I_{i,j}} \equiv \frac{1}{I_i} + \frac{1}{I_j}, \tag{1}$$

where K_i, I_i, ϕ_i are the spring constant of ith torsional bar, the moment of inertia of ith rigid disc, and the torsional angle of ith torsional bar. External disturbance on the ith disc is expressed as T_i. The moment of inertia of the left-end disc and the external disturbance on it are represented by I_0 and T_0.

Vibration energy in the real system propagates to the imaginary system based on propagation properties when Eq. (1) is satisfied. Elements whose suffixes exceed $(m+1)$ in Eq. (1) are virtual, so we compensate for the term relating to ϕ_{m+1} (shown $\underbrace{\ast}_{a}$ in Eq. (1)) as control acceleration. Previously [14], equations of motion including variables whose suffixes exceed $(m+1)$ were solved by online calculation, where variable ϕ_m (shown $\underbrace{\ast}_{b}$ in Eq. (1)) is measured.

In this paper, the imaginary system with impedance-matching characteristic at the end element is introduced, which gives a transfer function with no initialization. Because the transfer function uses a wave solution, the imaginary system is confined as a uniform system whose wave solution is known. The real system is not required to be uniform.

2.2. Wave-absorption condition

The ith equation of motion of the uniform system with no external disturbance and influence of boundary condition (called as an inner element) is

$$\ddot{\phi}_i - \omega_0^2\phi_{i-1} + 2\omega_0^2\phi_i - \omega_0^2\phi_{i+1} = 0 \quad (i \neq 1, 0), \quad \omega_0^2 = K/I. \tag{2}$$

Laplace transformation of Eq. (2) is

$$-\Phi_{i-1}(s) + (2 + s^2/\omega_0^2)\Phi_i(s) - \Phi_{i+1}(s) = 0, \tag{3}$$

where $\Phi_i(s)$ is Laplace transformation of ϕ_i. Substituting general solution $\Phi_i(s) = \beta(s)^i$ ($\Phi_{i+1}(s) = \beta(s)\Phi_i(s)$) into Eq. (3), we obtain the specific roots

$$\beta(s) = 1 + s^2/(2\omega_0^2) \mp s/\omega_0\sqrt{1 + s^2/(4\omega_0^2)} \equiv 1 + s^2/(2\omega_0^2) \mp s/\omega_0\sqrt{\beta^0(s)} \equiv \beta(s)^+, \beta(s)^- \tag{4}$$

and the general solution

$$\Phi_i(s) = c_1(s)(\beta^+)^i + c_2(s)(\beta^-)^i \equiv \Phi_i^+(s) + \Phi_i^-(s), \tag{5}$$

where $c_1(s)$ and $c_2(s)$ are arbitrary constants determined by boundary conditions.

Introducing $s = \mathrm{j}\omega$ (j is the imaginary unit), when $\beta^0(\mathrm{j}\omega)$ in Eq. (4) is positive, $\beta^+(\mathrm{j}\omega)$ represents a positive propagating solution to higer numbered elements and $\beta^-(\mathrm{j}\omega)$ a negative propagating solution to lower numbered elements. The condition of existence of propagating solution $0 \leqslant \beta^0(\mathrm{j}\omega)$ gives the limit frequency as

$$\omega \leqslant 2\sqrt{K/I} \equiv 2\omega_0. \tag{6}$$

Eq. (6) shows that the torsional bar-and-rigid disc wave-absorption controller must have a specific frequency $\omega_0 = \sqrt{K/I}$, i.e., greater than half of the disturbance frequency of the controlled system. From Eq. (4), we obtain

$$\left|\beta^{+(-)}\right| = 1 \tag{7}$$

which means that the steady-state wave amplitude is constant regardless of frequency.

Wave-absorption control at the boundary is simpler than elsewhere in the system because waves from outside of the boundary need not be considered. Impedance-matching control is a well-known wave-absorption strategy at the boundary [15,16], and so we study the impedance-matching condition for a lumped torsional system. Because the equation of motion for a uniform torsional bar-and-rigid disc is the same as that of a uniform spring-and-mass system (Fig. 2), we use the latter to make it easier to take impedance into consideration. We regard K,I, torsional spring constant and moment of inertia of torsional system, as k,m, spring constant and mass of spring-and-mass system mathematically.

The mechanical impedance for the positive propagating solution is defined as the ratio of the spring force between ith and $(i+1)$th masses to the velocity of ith mass as

$$\zeta^+(s) = K\frac{\Phi_i(s) - \Phi_{i+1}(s)}{s\Phi_i(s)} = K\frac{(1-\beta^+)}{s} = K\bar{\zeta}^+(s). \tag{8}$$

Substituting solution β^+ given by Eq. (4) into Eq. (8), we obtain

$$\zeta^+(s) = \sqrt{IK}\left\{-\mathrm{j}\omega/(2\omega_0) + \sqrt{1 - \omega^2/(4\omega_0^2)}\right\}. \tag{9}$$

Using Eq. (8), Eq. (3) is transformed as

$$(1 + s^2/\omega_0^2)\Phi_i - \Phi_{i-1} = \Phi_{i+1} - \Phi_i = (\beta^+ - 1)\Phi_i = -\bar{\zeta}^+ s\Phi_i. \tag{10}$$

When the right-hand term $-\bar{\zeta}^+ s\Phi_m$, putting $i = m$ in Eq. (10), is used as an impedance-matching wave control force at the end element, the equation of motion of the end element should satisfy Eq. (11) to be in a positive

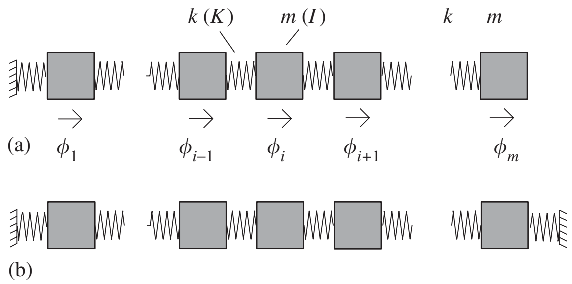

Fig. 2. Uniform mass-and-spring system equivalent to torsional system: (a) fixed–free boundary and (b) fixed–fixed boundary.

wave propagating state

$$(1 + s^2/\omega_0^2)\Phi_m - \Phi_{m-1} = 0. \tag{11}$$

The equation of motion of the end element of the free boundary condition (Fig. 2(a)) satisfies the above relation, but that of the fixed boundary condition (Fig. 2(b)) does not, shown as

$$(2 + s^2/\omega_0^2)\Phi_m - \Phi_{m-1} = 0. \tag{12}$$

From Eq. (1), the end element of a uniform torsional system corresponds to Eq. (12), so we must consider the generalized impedance-matching condition of wave absorption applied to the end element of the fixed boundary condition. The reason why an uniform torsional vibration system, with both ends free, corresponds to a fixed–fixed spring-and-mass system is that Eq. (1) is represented by a torsional rather than a rotation angle, that is, rigid rotation disappears in Eq. (1). As seen from the above consideration, the wave-absorption condition at the end element is obtained by making wave solution β^+ obtained from the equation of motion of an inner element satisfy that of the end element. Taking this into consideration, Eq. (10), also putting $i = m$, is transformed as

$$\left(2 + s^2/\omega_0^2\right)\Phi_m(s) - \Phi_{m-1}(s) = (-\bar{\zeta}^+ + 1/s) \cdot s\Phi_m(s) = \beta^+\Phi_m(s). \tag{13}$$

Since the left-hand side of Eq. (13) is the same as Eq. (12), Eq. (13) represents the equation of motion of the end element when $(-\bar{\zeta}^+ + 1/s)s\Phi_m(s)$ is the control term. Substituting relation $\beta^+\Phi_m(s) = \Phi_{m+1}(s)$ into Eq. (13) apparently gives the same relation of three adjacent elements as the equation of motion of an inner element.

The wave-absorption condition on an arbitrary element is therefore equivalent to satisfying the wave progressive solution in its equation of motion. The so-called usual impedance-matching condition is a specific case of the end element of the free boundary condition.

As Eqs. (10) and (13) show, it is more convenient for the lumped torsional system considered here to deal with the relation of control force to displacement rather than to velocity, i.e., stiffness or compliance rather than impedance.

Eqs. (10) and (13) give two wave-absorption strategies for a lumped torsional system—one to apply the generalized impedance-matching condition (GIP) given by Eq. (13) and the other to introduce an imaginary system with an impedance-matching condition at the end (IIP) where the free boundary condition is conveniently used. GIP is better than IIP method for a uniform controlled system, because a smaller order controller is constructed. When the controlled system is nonuniform, however, we must introduce an imaginary system to construct a virtual uniform system. In this case, a 1 dof imaginary system with

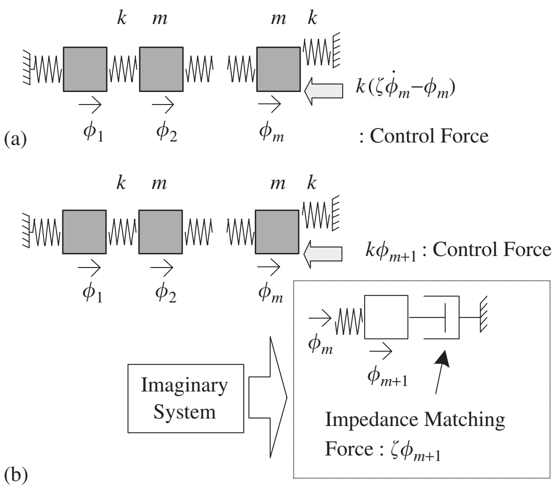

Fig. 3. GIP and IIP method: (a) GIP and (b) IIP.

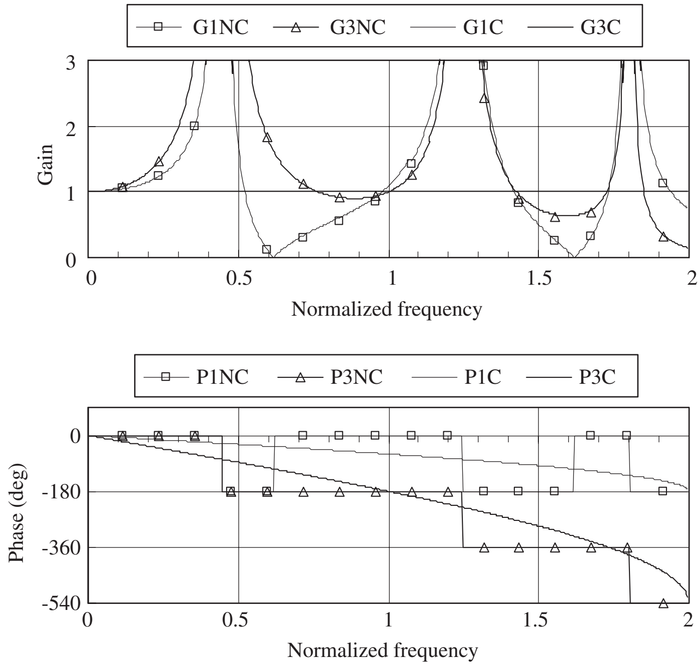

Fig. 4. Response of 3 dof mass-and-spring system: G1(3)NC, P1(3)NC, gain and phase of element 1(3) with no control and G1(3)C, P1(3)C, gain and phase of element 1(3) with wave-absorption control.

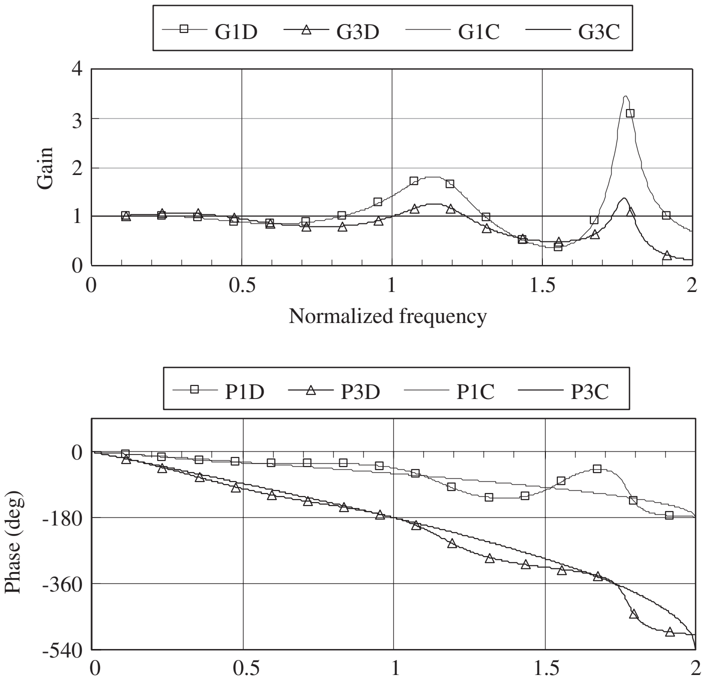

Fig. 5. Response of 3 dof mass-and-spring system with normal damping at end element (damping coefficient $= \sqrt{IK}$): G1(3)D, P1(3)D, gain and phase of element 1(3) with normal damping and G1(3)C, P1(3)C, gain and phase of element 1(3) with wave-absorption control.

impedance-matching characteristic constructs a wave-absorption system because a 2 dof uniform system constructed by 1 real and 1 dof imaginary systems satisfy Eq. (11) regarding m as $m+1$. If a nonuniform controlled system has two same end-side elements, we use GIP because Eq. (13) is satisfied. Again, wave-absorption control for a lumped system can be constructed for any system at the end element. Fig. 3 shows these strategies schematically.

Fig. 4 shows the response of a 3 dof uniform spring-and-mass system for a sinusoidal displacement input at the left end element 1 together with wave absorption control at the right end element 3 and without control.

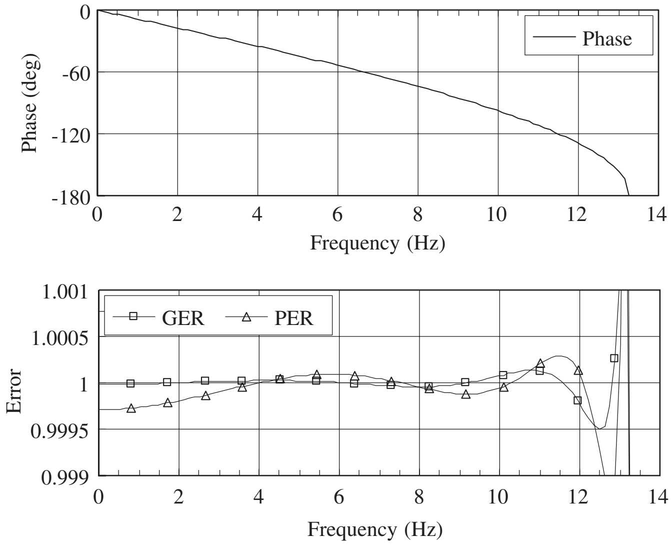

Fig. 6. Transfer function of GIP (β^+): phase: phase of Eq. (4); GER, PER, approximation error of gain and phase (GER = GAP/G, PER = PAP/P; GAP, approximated gain, PAP, approximated phase).

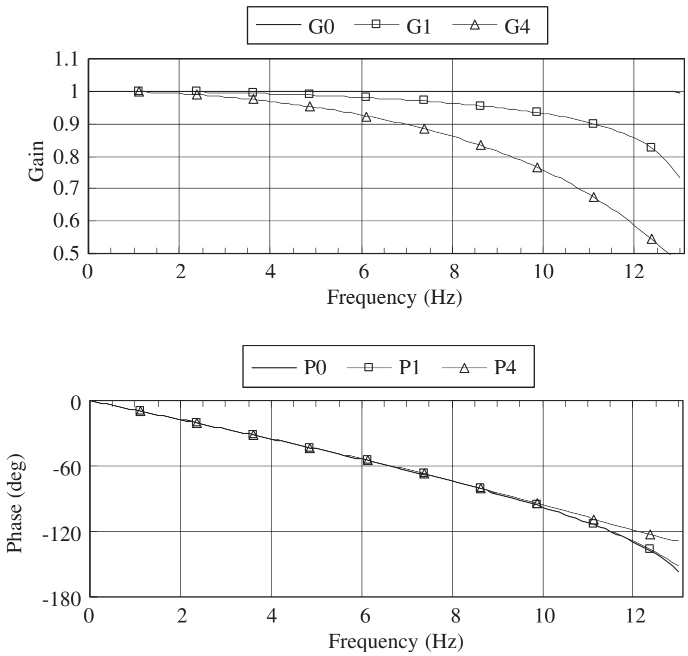

Fig. 7. Transfer function of GIP controller for sampling time 1 and 4 ms: G0, G1, G4, gain of continuous system, digital filter with sampling time 1 and 4 ms; P0, P1, P4, phase of continuous system, digital filter with sampling time 1 and 4 ms.

The controlled amplitude remains constant corresponding to the input amplitude up to wave propagation limit frequency ($\omega/\omega_0 = 2$). The controlled phase of element 1 changes from 0° to −180° and element 3 from 0 to −540° up to the wave propagation limit frequency. The uncontrolled phases of elements 1 and 3 also change from 0° to −180° and 0° to −540° stepwise. Here, the uncontrolled phase is depicted as continuous functions, unusual in vibration textbooks, which helps in understanding wave-controlled characteristics—phase shift characteristics inherently exist in the spring-and-mass vibration system. The uncontrolled phase and amplitude approach the wave controlled one if a velocity-dependent normal damping is introduced in place of wave-absorption control (Fig. 5), with damping coefficient equivalent to $\zeta = \sqrt{IK}$ used.

2.3. Wave-absoption control filter

The wave-absorption control filter is designed for a uniform torsional system whose parameters are the same as those of the experiment detailed in the next section. Characteristic root $\beta^+(s)$ is approximated by

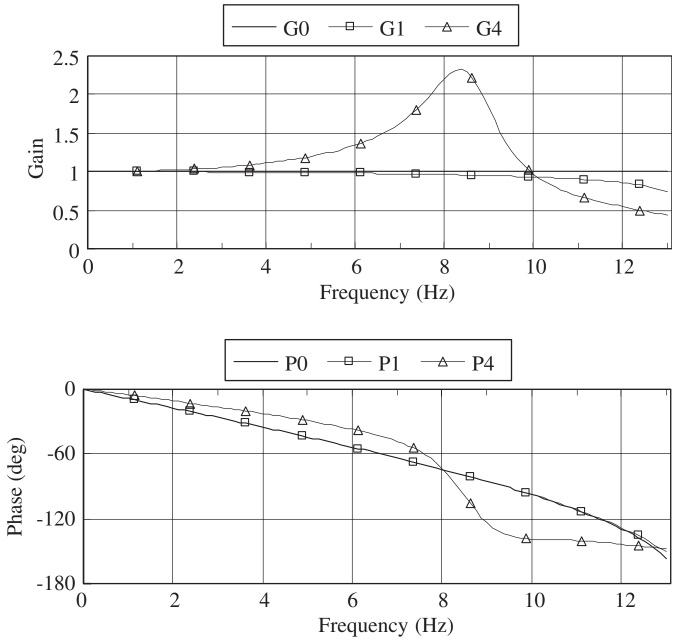

Fig. 8. Transfer function of IIP controller for sampling time 1 and 4 ms: G0, G1, G4, gain of continuous system, digital filter with sampling time 1 and 4 ms; P0, P1, P4, phase of continuous system, digital filter with sampling time 1 and 4 ms.

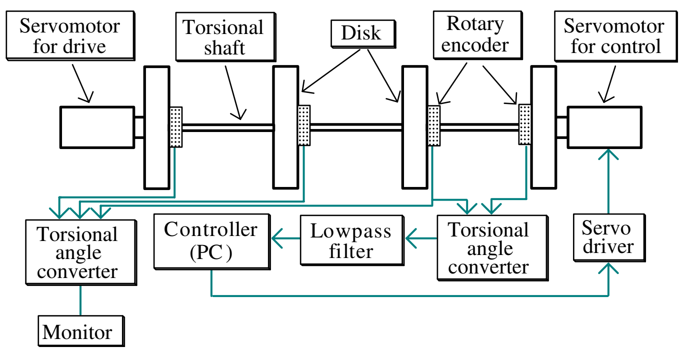

Fig. 9. Experimental apparatus.

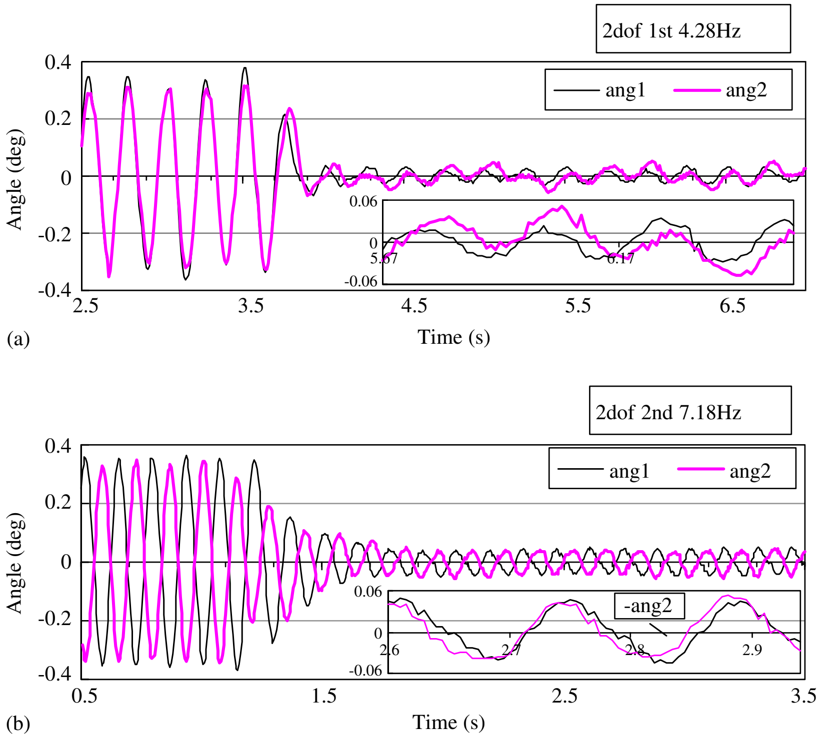

Fig. 10. Response of controlled and uncontrolled amplitudes of 2 dof uniform system by GIP: (a) at the first resonance frequency and (b) at the second resonance frequency.

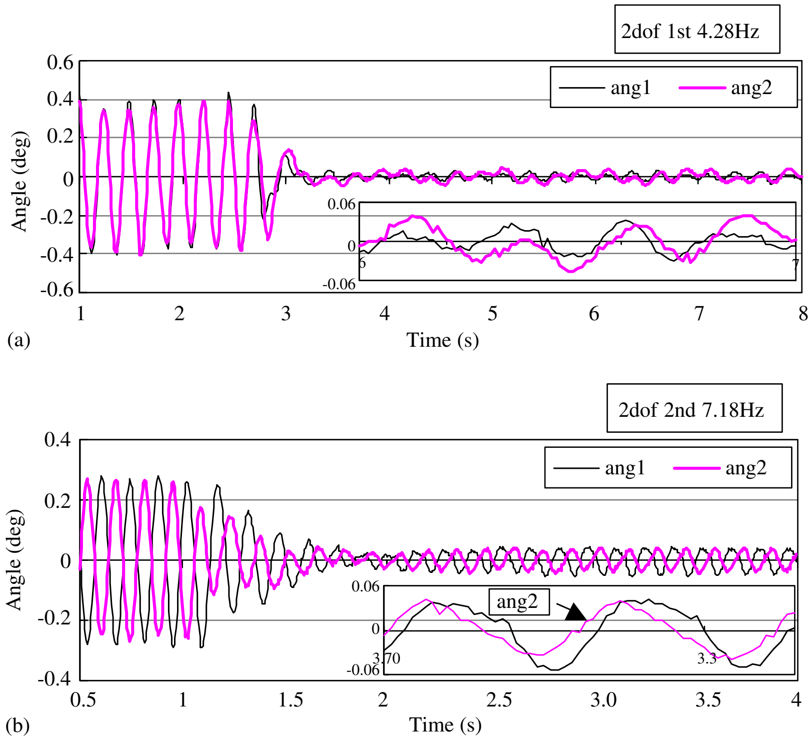

Fig. 11. Response of controlled and uncontrolled amplitudes of 2 dof uniform system by IIP: (a) at the first resonance frequency and (b) at the second resonance frequency.

six-order fractional polynomials. Fig. 6 shows $\beta^+(s)$ and the approximated representation for frequency ω using the spring constant of torsional bar $K = 21.4$ Nm and moment of inertia $I = 0.0123\,\mathrm{kgm}^2$ ($\omega_0 = 6.64$ Hz). Gain is constant at unity and the phase changes from 0 to $-180°$. The approximated function is obtained by the *invfreqs* function of MATLAB, which agrees well with original function $\beta^+(j\omega)$ in all frequency ranges except for the vicinity of wave propagation limit frequency $2\omega_0$. By zero-order hold z-transformation of this approximated transfer function with sampling periods 1 and 4 ms, an IIR digital control filter for GIP is obtained (Fig. 7). This transfer function was calculated by using the *freqz* function of MATLAB.

Similarly, an imaginary system with impedance-matching condition is designed as an IIR digital filter. Because the 1 dof imaginary system having a free end condition is sufficient for impedance-matched wave absorption, the transfer function is expressed as

$$K\frac{\Phi_{m+1}(s)}{\Phi_m(s)} = K\left[1 + s^2/\omega_0^2 + (s/\omega_0)\zeta^+(s)/\sqrt{IK}\right]^{-1}. \tag{14}$$

By zero-order hold z-transformation of Eq. (14) with sampling period 1 and 4 ms, we obtain an IIR digital filter (Fig. 8) with approximation errors at $\omega_0 \leqslant \omega$ for the 4 ms sampling period. Since no characteristics of

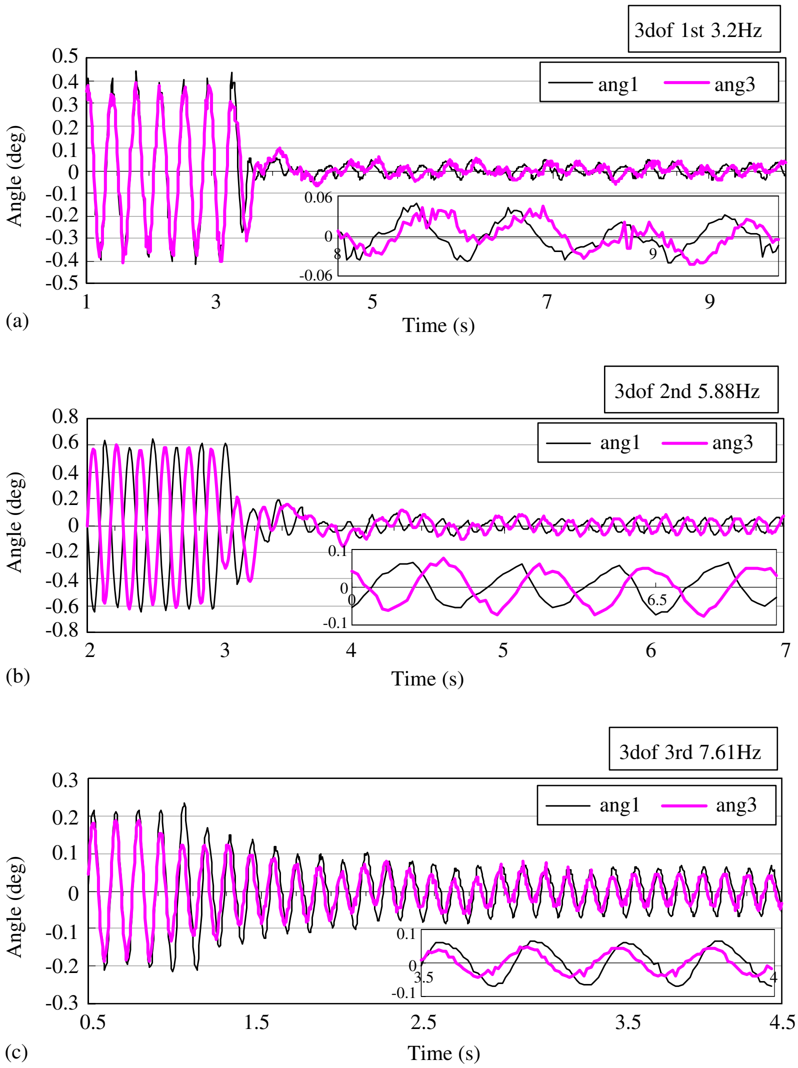

Fig. 12. Response of controlled and uncontrolled amplitudes of 3 dof uniform system by GIP: (a) at the first resonance frequency, (b) at the second resonance frequency and (c) at the third resonance frequency.

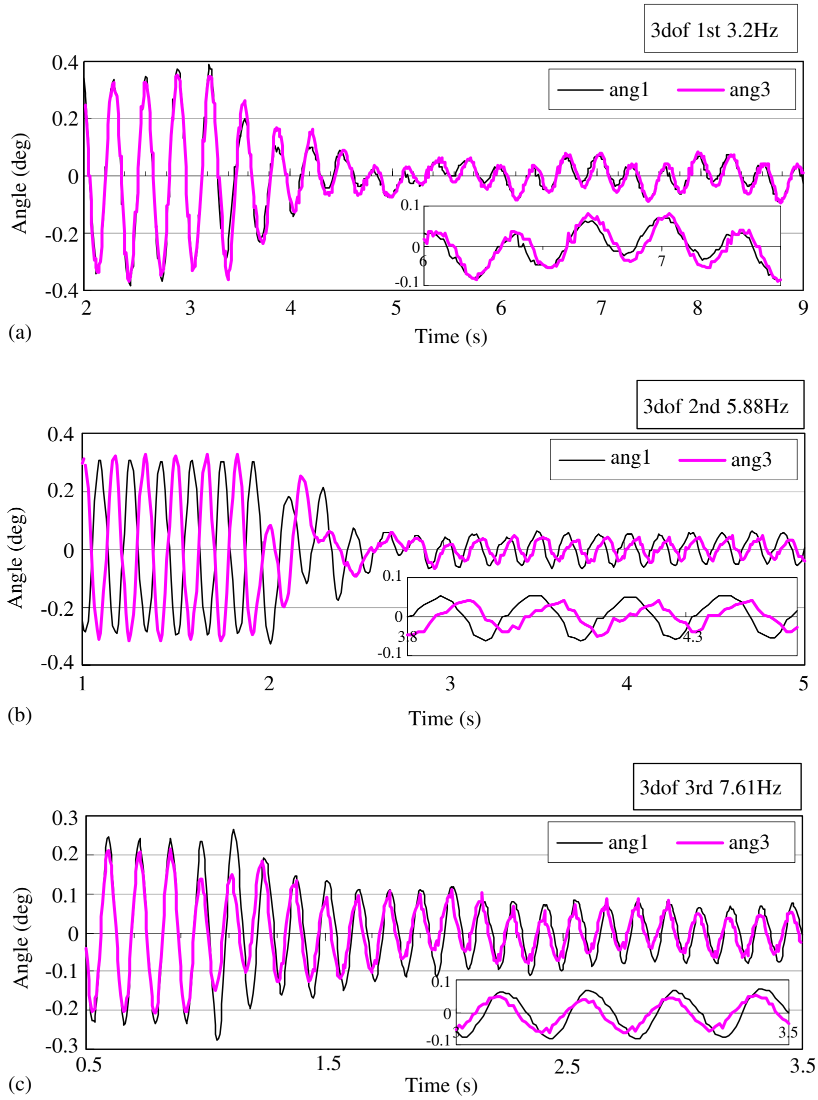

Fig. 13. Response of controlled and uncontrolled amplitudes of 3 dof uniform system by IIP: (a) at the first resonance frequency, (b) at the second resonance frequency and (c) at the third resonance frequency.

such errors are seen in Fig. 6, this may be due to resonance phenomena of the 1 dof imaginary system resembling characteristics in Fig. 5. By losing the wave propagation compensation characteristic, the roughly approximated controller approaches normal damping.

3. Experiments

We have conducted experiments by using the IIR filters of GIP and IIP (Fig. 9). The uniform controlled vibration system consists of discs 200 mm in diameter and 20 mm thick and torsional bars 4 mm in diameter and 100 mm long for 2 and 3 dof uniform systems. AC servomotors are used for drive and control. The drive motor has a rated torque of 0.9 Nm and a rated speed of 3000 rev/min and the control motor has a rated torque of 0.16 Nm and a rated speed of 3000 rev/min. The measurement system consists of rotary encoders, a torsional angel converter, low- and high-pass filters, and a personal computer (CPU clock: 333 MHz). Torque disturbance is applied by torque fluctuation of the AC drive motor in constant speed mode, which is 4 times, 2 times, and 1 time per rotation, so, resonance may occur at a rotation speed of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{1}$ of the natural

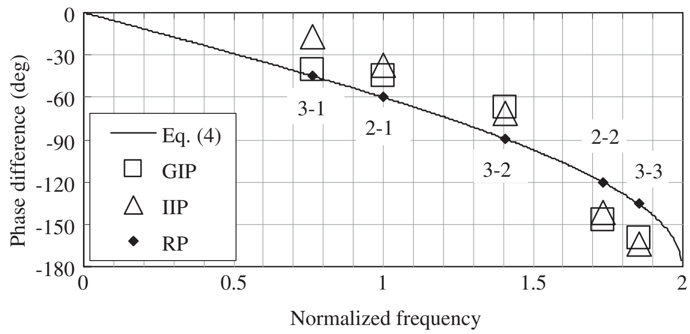

Fig. 14. Wave propagation characteristics in controlled system; □, experimental result of GIP filter; △, experimental result of IIP filter; ♦, resonance point (*m*–*n*: *n*th resonance of *m* dof system).

frequency. Torque magnitude cannot be adjusted. Experiments are conducted for 2 and 3 dof with a sampling period of 4 ms.

Figs. 10 and 11 show controlled and uncontrolled amplitudes of 2 dof uniform system by GIP and IIP, with (a) at the first resonance frequency and (b) at the second resonance frequency. Amplitudes are suppressed considerably by control and phase shifts between ϕ_1 and ϕ_2 of the controlled amplitudes are seen. Figs. 12 and 13 show controlled and uncontrolled amplitudes of 3 dof uniform system by GIP and IIP, with (a) at the first resonance frequency, (b) at the second resonance frequency, and (c) at the third resonance frequency. Amplitudes are suppressed considerably by control and the phase between ϕ_1 and ϕ_3 of controlled amplitudes is shifted. These results are greatly superior to those produced by online simulation of the imaginary system in Ref. [14] (Appendix A), which confirm the high control performance of the developed wave-absorption control filter. Fig. 14 shows wave propagation characteristic β^+ in the controlled system. The experimental phase shift is obtained by using a band-pass filter at the corresponding frequency. The phase shift for the 3 dof system is calculated as half of the shift between ϕ_1 and ϕ_3. The observed phase shift angle between adjoining elements agrees well with Eq. (4)—further evidence of the wave control performance of the developed control filter. Although control performance between GIP and IIP differs in Figs. 7 and 8, we can see no marked difference in experiments (Figs. 10–14).

4. Conclusions

We have developed wave-absorption control filters for the end element of a lumped torsional system and have demonstrated its effectiveness in experiments, with the following major results:

(1) The wave-absorption filter for the end element of a lumped torsional system has been developed by using the characteristic root of the equation of motion of an inner element with no influence of boundary condition. It is quite useful for a uniform controlled system experimentally, and it is also applicable to a nonuniform system with two same end elements.
(2) The wave-absorption filter for the end element of a lumped torsional system constructed by using the 1 dof imaginary system and impedance-matching condition is useful for both a uniform system and a nonuniform system.
(3) The impedance-matching condition of wave-absorption control at the end element is a specific condition for a free end boundary condition, and the general wave-absorption condition is to satisfy a characteristic root on the equation of motion of the end element.
(4) The wave-absorption control filter constructed using only the characteristic root of the equation of motion of an inner element is more suitable than that using the characteristic root and the imaginary system because of the lesser order of the transfer function with practically no loss of control performance.
(5) The characteristic root of wave propagation is usefully approximated by six-order fractional polynomials to construct digital control filters.

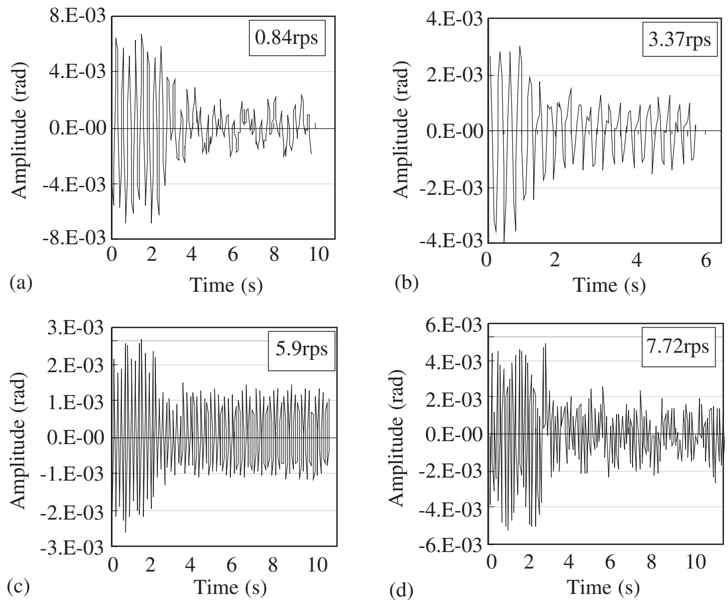

Fig. A1. Timing chart of 3 dof real system experiment at a speed of (a) a quarter of first resonance, (b) first resonance, (c) second resonance, and (d) third resonance.

Appendix A

Experimental results of wave absorption control with online simulation of imaginary system [14] are shown below for comparison with those in this paper (Fig. A1).

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